

# MH370 Burst Frequency Offset Analysis and Implications on Descent Rate at End-of-Flight

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## I. INTRODUCTION

ON 7 March 2014 at 16:41:43Z<sup>1</sup>, Malaysian Airlines flight MH370 departed Kuala Lumpur (KL) International Airport bound for Beijing. Less than an hour later, following the last recorded radio transmission from MH370 at 17:19:30Z, the plane's secondary radar transponder went offline. As evidenced by Malaysian military radar, the plane (registration number 9M-MRO) then veered off course unexpectedly, backtracked across the Malaysian Peninsula, and was then tracked heading northwest from Penang through the Malacca Straits. After disappearing from radar at 18:22:12Z, it re-established a satellite communications (SATCOM) link with the Inmarsat satellite I-3F1 at 18:25:27Z. By analyzing a series of automated messages exchanged via that satellite between the plane and an Inmarsat ground station in Perth, Australia, it was determined that the plane continued to fly for six hours, before finally ceasing message exchange with the ground station at 00:19:37Z on 8 March 2014. This article discusses specifically the analysis of burst frequency offset (BFO) metadata from the SATCOM messages. Importantly, it is shown that the BFOs corresponding to the last two SATCOM messages from the plane at 00:19:29Z and 00:19:37Z suggest that flight MH370 was rapidly descending and accelerating downwards when message exchange with the ground station ceased.

An initial analysis by Inmarsat of the SATCOM metadata for MH370 in the last six hours of flight suggested that MH370 had flown into the Southern Indian Ocean before SATCOM was ultimately lost (see [1] for further details). As summarized in [2] an intensive aerial and surface search was undertaken in the Southern Indian Ocean by an international search team during March and April 2014, with no MH370 related debris found. On 28 April 2014, the aerial search concluded and the search transitioned to an underwater phase [2]. The Australian Transport Safety Bureau (ATSB) took responsibility for the definition of the underwater search zone. It convened an international flight path prediction working group bringing together experts in satellite communications, statistical data processing and aviation, in order to estimate the most likely final location of flight MH370. The group consisted of representatives from the Australian Defence Science and Technology (DST) Group and the other organizations listed in the Acknowledgment section of this article.

New methods of analyzing the Inmarsat data were developed by the group, resulting in the release of reports

concerning the likely final location of flight MH370 from the ATSB in August 2014 [3], October 2014 [4], and December 2015 [5]. Inmarsat also published an article regarding their contribution to the flight path reconstruction effort [1]. The DST Group contribution that assisted in the definition of an extended priority search area in December 2015 [5] has been detailed in [2]. This demonstrated how Bayesian analysis was used to identify a high probability region of where the plane was believed to be at the time of last SATCOM transmission (00:19:37Z 8 March 2014). The DST Group Bayesian method used a prior probability distribution defined by the Malaysian military radar, a likelihood function describing the relationship between SATCOM measurements and the aircraft position and velocity during the flight, and a model of the aircraft dynamics.

It should be noted that this article does not cover the Bayesian method used in defining the underwater search area. For details on that method, the reader is referred to [2]. Instead, this article focuses on:

- 1) A brief review of the statistical analysis of BFOs for several previous flights of 9M-MRO.
- 2) Examination of the effects of the plane's vertical velocity on the BFO.
- 3) An analysis of the effect of the track angle of the plane on the BFO towards the end-of-flight.
- 4) The establishment of a BFO trend throughout the last 6 hours of flight so as to determine an expected BFO at end-of-flight.
- 5) An analysis of the behavior of the frequency oscillator in the plane's satellite data unit (SDU) after power outage events such as those believed to have occurred twice during flight MH370, and how this affects the BFO.
- 6) An analysis that shows that the final two BFO's are consistent with MH370 being in a rapid descent and accelerating downwards.

The remainder of this article is structured as follows. Section II presents a brief timeline of key events during the MH370 accident flight. In Section III, a review of the SATCOM model is provided, along with a brief review of the BFO statistics. This serves as a summary of work previously presented in [2]. Section IV describes the effects of aircraft position and velocity on the BFO. The settling behavior of the SDU's frequency oscillator for 9M-MRO after power-up is then detailed in Section V. This is important because the SDU is believed to have undergone a power outage between 00:11Z and 00:19Z on 8 Mar 2014, immediately preceding the last two SATCOM transmissions from MH370. The effect of oscillator

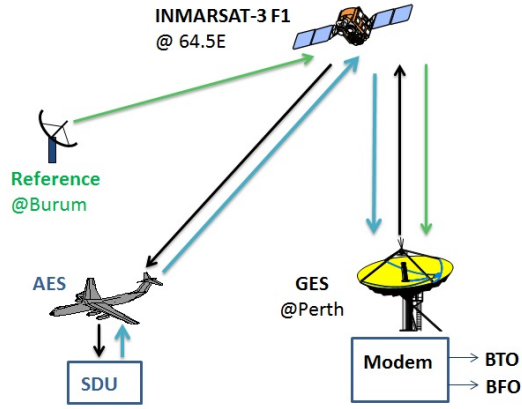


Fig. 1. System model of the satellite communication system [2].

warm-up on the BFO after power-up can be used in bounding the descent rates for MH370 at 00:19:29Z and 00:19:37Z. In Section VI an analysis of the descent rate of MH370 based on the last two BFO values is presented. This analysis derives lower and upper bounds on the descent rate at 00:19:29Z and 00:19:37Z. Conclusions are presented in Section VII.

## II. TIMELINE OF EVENTS DURING THE ACCIDENT FLIGHT

A comprehensive description of the events occurring during the MH370 accident flight is provided in [6]. A summary of salient events to this article is provided in Table I.

## III. REVIEW OF SATCOM MODEL

The accident aircraft was fitted with a SATCOM terminal that used the Inmarsat Classic Aero system [1], which uses geosynchronous satellites to relay messages between aircraft and ground stations. During the flight, messages were passed between the aircraft and a ground receiving station located in Perth, Australia, via the Inmarsat-3F1 satellite. Figure 1 illustrates the SATCOM system in use during the flight. The aircraft is referred to as the Aircraft Earth Station (AES) and the ground receiving unit is referred to as the Ground Earth Station (GES). Inmarsat-3F1 is a satellite in geosynchronous orbit at 64.5° East longitude and it was used exclusively for the duration of the flight.

An AES is equipped with an SDU comprising a satellite modem with auxiliary hardware and software. Transmission of data over the satellite is via bursts which are scheduled to arrive at the GES at a specified time and a given frequency. As explained in [2], [1], communications from multiple users are coordinated by the allocation of different time and frequency slots to each user. This is done without knowledge of individual AES locations or precise knowledge of the satellite location. Therefore, messages from a given AES might not arrive at the GES at exactly the expected time, and generally would arrive slightly later. The difference between the expected time of arrival (based on a nominal assumed position for the satellite and the AES) and the actual time of arrival is referred to as the Burst Timing Offset (BTO). The

BTO is a measure of how far the aircraft is from the sub-satellite position<sup>2</sup>.

The relative velocity between the satellite and the AES, as well as between the satellite and the GES, leads to a Doppler frequency offset on the signals received at the GES. Coupled with small frequency offsets inherent in the reference frequency oscillators in the AES, satellite and GES, this results in a net difference between the expected and actual frequency of the signal presented to the modem in the GES for a given user. Frequency compensations applied onboard the aircraft (aircraft induced Doppler pre-compensation) and at the ground station (Enhanced Automatic Frequency Correction, which utilizes the reference signal transmitted from a reference station in Burum, Netherlands), [2], [1] serve to reduce the possible difference between the expected and actual frequency of the messages received from the aircraft. The residual difference between the expected frequency of each communications burst and the actual received frequency is referred to as the BFO.

### A. Review of BFO Statistics

Based on 20 previous flights of 9M-MRO in the week leading up to the accident flight (see [2] for further details), a histogram was produced for the difference between the predicted BFO (based on known details of the plane and the satellite's position and velocity) and the measured BFO (based on Inmarsat ground station logs). This difference (i.e. predicted minus measured) is referred to as the BFO error. The histogram of the BFO error is shown in Fig. 2, along with a Gaussian distribution fit line. It can be seen that the distribution is somewhat Gaussian. The standard deviation of the BFO error was found in [2] to be 4.3 Hz. Whilst it is reasonable to apply bounds on the possible BFO error based on  $\pm 3$  standard deviations as was done for the approximate analysis described in [7], for the purpose of the descent analysis presented later in Sec. VI, it is assumed the BFO error is strictly bounded on the larger interval  $[-28, +18]$  Hz, which corresponds to the bounds of all 2501 observed valid<sup>3</sup> in-flight BFO error values available from the preceding 20 flights of 9M-MRO.

## IV. EFFECTS OF AIRCRAFT POSITION AND VELOCITY ON THE BURST FREQUENCY OFFSET

### A. Mathematical Description of the BFO

In [2], the BFO is defined mathematically at time step  $k$  as the sum of a noiseless component  $h_k^{\text{BFO}}$  and a scalar  $w_k^{\text{BFO}}$  that represents the BFO noise. The noiseless component of the BFO is defined in [2] as the sum:

$$h_k^{\text{BFO}}(\mathbf{x}_k, \mathbf{s}_k) = \Delta F_k^{\text{up}}(\mathbf{x}_k, \mathbf{s}_k) + \Delta F_k^{\text{down}}(\mathbf{s}_k) + \delta f_k^{\text{comp}}(\mathbf{x}_k) + \delta f_k^{\text{sat}}(\mathbf{s}_k) + \delta f_k^{\text{AFC}}(\mathbf{s}_k) + \delta f_k^{\text{bias}}(\mathbf{x}_k, \mathbf{s}_k), \quad (1)$$

where

- $\mathbf{x}_k$  denotes the state vector of the aircraft;
- $\mathbf{s}_k$  denotes the state vector of the satellite;

<sup>2</sup>The sub-satellite position is the point on the earth directly below the satellite.

<sup>3</sup>One outlier was removed as explained in Sec. V.

TABLE I  
TIMELINE OF EVENTS DURING MH370

Date and Timestamp(s)	Description	Comments
7 Mar. 2014 16:42Z	MH370 departs Kuala Lumpur Airport	Normal take-off
Last ACARS transmission	7 Mar. 2014 17:07Z	Everything normal, on path to Beijing
Loss of secondary radar	7 Mar. 2014 17:21:13Z	Onboard transponder goes inactive; last civilian radar
7 Mar. 2014 18:22:12Z	Last Malaysian military radar contact	MH370 shown tracking NW through Malacca Straits
7 Mar. 2014 18:25Z-18:28Z	Satellite Data Unit (SDU) log-on sequence	If flying level, MH370 still tracking NW
7 Mar. 2014 18:39-18:41Z	Unanswered ground-to-air phone call	If flying level, MH370 tracking southwards
7 Mar. 2014 19:41Z to 8 Mar. 2014 00:11Z	SATCOM metadata approx. hourly from MH370	[1], [2] suggest flight into Southern Indian Ocean
8 Mar. 2014 00:19:29Z-00:19:37Z	Partial SDU log-on sequence	Implies rapid descent

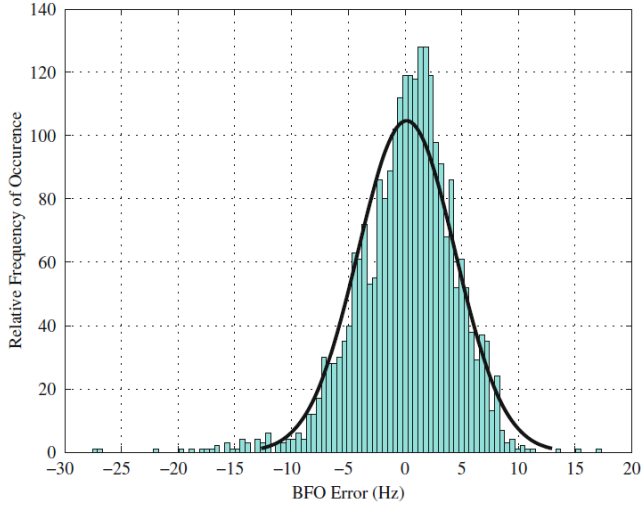


Fig. 2. Histogram of BFO errors for 20 flights of 9M-MRO prior to MH370 (reproduced from Fig. 5.5 of [2]). The mean and standard deviation for the distribution are 0.18 Hz and 4.3 Hz, respectively [2].

- $\Delta F_k^{\text{up}}(\mathbf{x}_k, \mathbf{s}_k)$  is the uplink (aircraft to satellite) Doppler shift;
- $\Delta F_k^{\text{down}}(\mathbf{s}_k)$  is the downlink (satellite to ground station) Doppler shift;
- $\delta f_k^{\text{comp}}(\mathbf{x}_k)$  is the frequency compensation applied by the aircraft;
- $\delta f_k^{\text{sat}}(\mathbf{s}_k)$  is the variation in satellite translation frequency;
- $\delta f_k^{\text{AFC}}(\mathbf{s}_k)$  is the frequency compensation applied by the ground station receive chain;
- $\delta f_k^{\text{bias}}(\mathbf{x}_k, \mathbf{s}_k)$  is a slowly varying bias due to errors in the aircraft and satellite oscillators and processing in the SDU.

By treating the bias  $\delta f_k^{\text{bias}}(\mathbf{x}_k, \mathbf{s}_k)$  as a constant determined at the source tarmac for any particular flight, as was done in [2] for MH370, any small time-varying component of the bias during a particular flight can be considered as part of the BFO noise (indeed this was done when compiling the results used to obtain the BFO error histogram shown in Fig. 2). Details regarding the terms  $\delta f_k^{\text{sat}}(\mathbf{s}_k)$  and  $\delta f_k^{\text{AFC}}(\mathbf{s}_k)$  are provided in [1]. Tabulated values of the sum of these two terms were provided by Inmarsat to the MH370 Flight Path Reconstruction Group to use in estimating the likely trajectory flown. These

two terms depend on the satellite state  $\mathbf{s}_k$  only, and not on the aircraft state  $\mathbf{x}_k$ . Moreover, the downlink Doppler  $\Delta F_k^{\text{down}}(\mathbf{s}_k)$  does not depend on the location or velocity of the aircraft, and can be calculated given the known frequency of the downlink and the known satellite state at any given time.

Equation (1) can be then be simplified to:

$$h_k^{\text{BFO}}(\mathbf{x}_k, \mathbf{s}_k) = \Delta F_k^{\text{up}}(\mathbf{x}_k, \mathbf{s}_k) + \delta f_k^{\text{comp}}(\mathbf{x}_k) + \delta f_k^{\text{det}}(\mathbf{s}_k), \quad (2)$$

where  $\delta f_k^{\text{det}}(\mathbf{s}_k)$  is effectively a known deterministic value for any time step  $k$ . The other terms in (2) couple the aircraft state  $\mathbf{x}_k$  by way of the aircraft position and velocity to the BFO as per the following equations adapted from [2]<sup>4</sup>:

$$\Delta F_k^{\text{up}}(\mathbf{x}_k, \mathbf{s}_k) = \frac{F^{\text{up}}(\mathbf{v}_s - \mathbf{v}_x)^{\text{T}}(\mathbf{p}_x - \mathbf{p}_s)}{c |\mathbf{p}_x - \mathbf{p}_s|}, \quad (3)$$

$$\delta f_k^{\text{comp}}(\mathbf{x}_k) = \frac{F^{\text{up}}(\hat{\mathbf{v}}_x)^{\text{T}}(\hat{\mathbf{p}}_x - \hat{\mathbf{p}}_s)}{c |\hat{\mathbf{p}}_x - \hat{\mathbf{p}}_s|}, \quad (4)$$

where the dependence on the time step  $k$  on the right hand side of the equations has been removed for simplicity of notation. In equations (3) and (4),  $|\cdot|$  is the three dimensional Cartesian distance, and:

- $F^{\text{up}}$  is the uplink carrier frequency;
- $c$  is the speed of light;
- $\mathbf{v}_s$  is the velocity vector of the satellite;
- $\mathbf{v}_x$  is the velocity vector of the plane;
- $\mathbf{p}_s$  is the position vector of the satellite;
- $\mathbf{p}_x$  is the position vector of the plane;
- $\hat{\mathbf{v}}_x$  is the SDU's estimate of the plane's velocity vector, which is obtained using the plane's track angle and ground speed, whilst assuming the vertical speed is zero;
- $\hat{\mathbf{p}}_s$  is the SDU's estimate of the position vector of the satellite, which assumes the satellite is at its nominal orbital slot of 0 degrees North and 64.5 degrees East;
- $\hat{\mathbf{p}}_x$  is the SDU's estimate of the position vector of the plane, which is obtained using the plane's latitude and longitude, whilst assuming the plane is at sea level.

#### B. Effect of uncompensated vertical velocity

The vertical speed of the plane is not used in the SDU Doppler compensation. As such, there is a direct contribution

<sup>4</sup>Note that the sign convention used in [2] is opposite to that used in (3)

of Doppler due to the proportion of the vertical velocity vector projected onto the radial direction from the aircraft to the satellite. It is straightforward to understand that if the plane was directly below the satellite, the vertical velocity vector would be fully towards or away from the satellite if the plane was climbing or descending. The direct contribution of the Doppler to the BFO in that case would be governed by the following standard Doppler equation.

$$\Delta F_k^{\text{up}}(\mathbf{x}_k, \mathbf{s}_k) = \frac{v_z \cdot F^{\text{up}}}{c}, \quad (5)$$

where  $F^{\text{up}}$  and  $c$  are as previously defined, and  $v_z$  is the vertical speed of the plane. Substituting an uplink frequency of 1646.6525 MHz (the uplink frequency stated in [1]) and a vertical velocity of 100 feet per minute (fpm), equivalent to 0.508 meters per second, equation (5) implies that the predicted BFO would increase by 2.8 Hz per 100 fpm of climb rate or decrease by 2.8 Hz per 100 fpm of descent rate if the plane were directly below the satellite. This is the maximum possible contribution of the plane's climb or descent rate to the BFO. In the more general case, equation (5) is moderated by the sine of the elevation angle  $\theta$  from the aircraft to the satellite. This is expressed as

$$\Delta F_k^{\text{up}}(\mathbf{x}_k, \mathbf{s}_k) = \frac{v_z \cdot F^{\text{up}} \sin(\theta)}{c}. \quad (6)$$

As such, at 00:19Z (the time at which the plane crosses the 7<sup>th</sup> arc provided by Fig. 3)<sup>5</sup>, where the elevation to the satellite is 38.8 degrees, the contribution to the BFO of climb or descent rate is reduced to approximately +1.7 Hz or -1.7 Hz per 100 fpm respectively.

### C. Effect of track angle on BFO towards end-of-flight

Another factor that needs to be considered in interpreting the BFOs at 00:19Z is the track angle of the aircraft. In [2], Fig. 5.6, a set of curves were shown illustrating the relationship of the BFO error and the aircraft track angle at 18:39Z. That was done to illustrate that under the assumption of level flight during the unanswered telephone call period of 18:39Z-18:41Z, MH370 would have been tracking in a southerly direction. The same model can be used to examine the relationship between the BFO error and the track angle at 00:11Z, when MH370 crossed the 6<sup>th</sup> arc, just 8 minutes prior to the last messages were received from MH370.<sup>6</sup> To generate the curve of BFO error vs. track angle, it is assumed that the aircraft crossed this arc at 38.67S, 85.11E. A previous sensitivity analysis has revealed the curve is relatively insensitive to the actual crossing point. Also, calculations revealed that (as shown in Fig. 5.6 of [2] at 18:40Z) the track angle of the aircraft influences the BFO to a greater degree for a faster assumed ground speed. To assess the maximum possible effect of track angle on the BFO at 00:19Z, a ground speed of 500 kts is assumed, and the BFO error vs. track angle is then

<sup>5</sup>The arcs referred to in this paper and other MH370 literature are segments of the rings in Fig. 3.

<sup>6</sup>Note that the 6<sup>th</sup> arc crossing is considered here to isolate effects of ground track angle variation as opposed to vertical velocity, since the measured BFO at the 6<sup>th</sup> arc is still consistent with level flight.

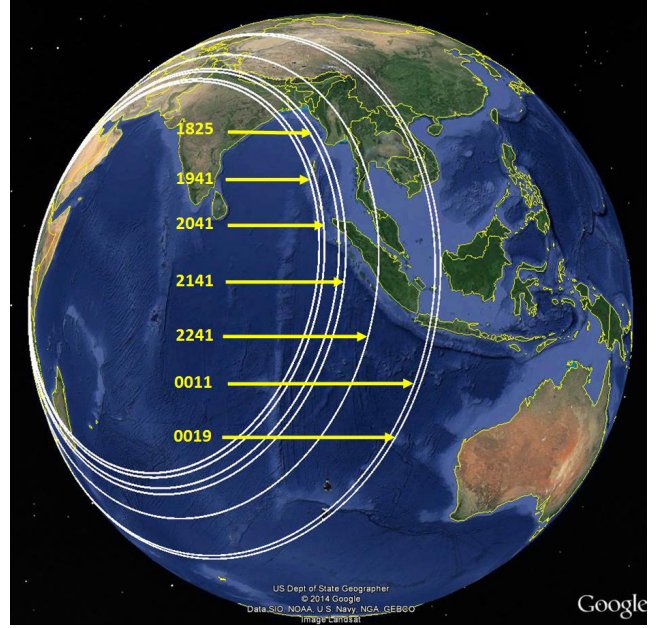


Fig. 3. BTO rings during the MH370 flight [3]. Note that the latitude of the arrows shown for each ring is arbitrary.

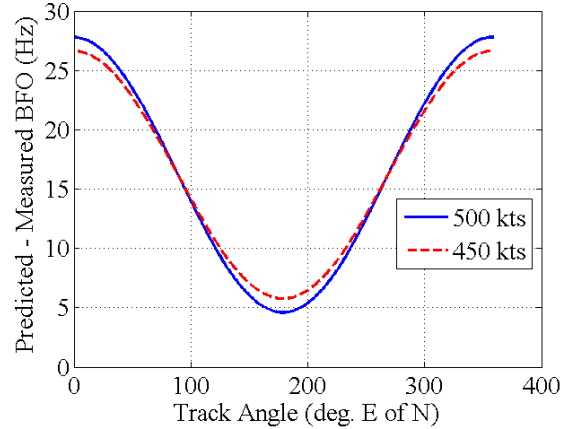


Fig. 4. BFO errors as a function of track angle at 00:11Z.

as shown in Fig. 4. Another curve assuming 450 kts ground speed is shown for comparative purposes. It is noted that the peak-to-peak variation of the BFO difference is similar for both considered ground-speeds.

### D. BFO Trend During the MH370 Flight

The measured BFOs from 19:41Z to 00:11Z are shown in Fig. 5 together with a line-of-best-fit. This line is extended forward to the time of the 00:19Z log-on resulting in an expected BFO of roughly 254 Hz<sup>7</sup>. With reference to the track angle curves presented in Fig. 4, the 00:11Z BFO error value for the southern-most track is roughly 6 Hz, meaning the measured BFO of 252 Hz was 6 Hz lower than the lowest value it could have been (assuming roughly level flight at

<sup>7</sup>Note the BFOs observed at 00:19Z were much lower than the expected value.

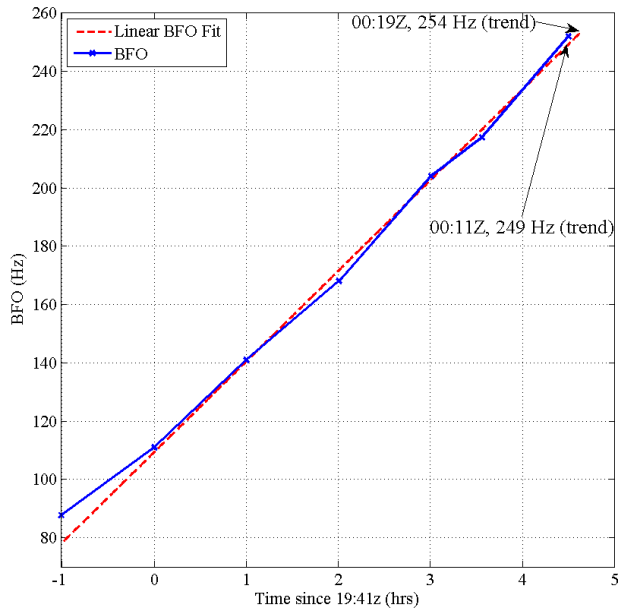


Fig. 5. Measured BFOs during the MH370 flight with trend-line extrapolated back to 18:40Z and forward to 00:19Z.

approximately 450 kts). If we assume the same error value at 00:19Z, and use the extrapolated BFO, assuming a south track with level flight and similar ground speed, the expected BFO value is 260 Hz.

The linear trend-line in Fig. 5 is also extended back by 1 hour to the time of the first unanswered satellite telephone call to MH370. The mean of the measured BFOs during that call attempt is also shown. Since the mean of the measured BFOs from the 18:39-18:41Z call attempt are in broad agreement with the linear trend observed in the BFOs from 19:41Z to 00:11Z (for which the BTOs themselves were consistent with straight and level flight [2]), this supports the finding in [2] that there were most-likely no major turns after the unanswered call attempt (see ([2], Fig. 10.5)).

## V. EFFECT OF SDU STARTUP ON THE BFO

The BFOs logged for messages from the plane received at the Perth GES at 00:19:29Z and 00:19:37Z 8 March 2014 were much lower than the expected value of 260 Hz. In the absence of any other factors affecting the BFOs at these times, this suggests a large uncompensated negative vertical velocity component in the BFOs. In determining how to interpret this data, it is important therefore to carefully consider any known other factors that could effect these BFOs. One potential factor is an SDU oscillator startup transient. This section analyses the range of expected effect on the BFO caused by a period of power outage followed by a restart of the SDU, as was a possibility between 00:11Z and 00:19Z on 8 March 2014 (see [3] for further details).

### A. Detailed Analysis of SDU Startup Effects on the BFO

At 18:25:27Z 7 March 2014, the Inmarsat GES in Perth received a SATCOM log-on request from 9M-MRO. A series of messages were exchanged in the following few minutes

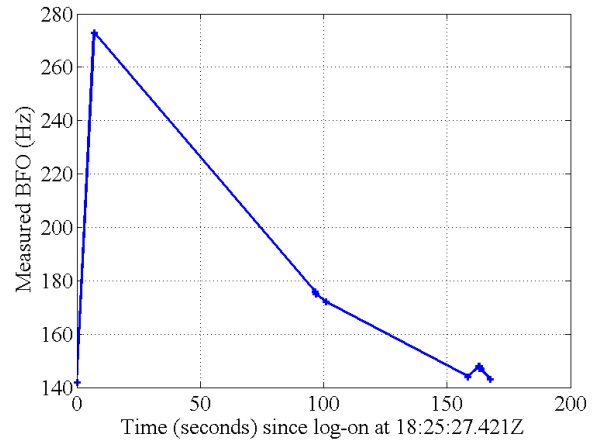


Fig. 6. Measured BFOs following the 18:25:27Z log-on message from MH370.

as part of a standard log-on sequence. The BFOs over those minutes displayed somewhat unusual behavior in that (barring the first BFO) the BFO appeared to be exhibiting a transient settling behavior. This is shown in Fig. 6. It was noted in [1] that the spike in BFO observed at this time<sup>8</sup> was not fully understood and whilst originally attributed to a possible aircraft maneuver it could also be related to actual frequency changes occurring during the logon sequence. A subsequent study [8] by the SATCOM sub-group of the MH370 Flight Path Reconstruction Group revealed this was most likely due to power-on frequency drift and subsequent stabilization of the oven-controlled crystal oscillator (OCXO) in the SDU. In [8], a number of different SDUs were tested to investigate the effects of power outages on the BFO during SDU power on, triggering a SATCOM logon such as that which occurred for MH370 at 18:25:27Z. It was found that whilst the frequency settling behavior was different for each individual SDU, any given SDU displayed repeatable behavior for a fixed outage duration, and similar settling characteristics for outages of different duration.

Based on sequences of Inmarsat BFO logs for 9M-MRO concerning four log-ons after periods of SATCOM outage in the week leading up to the accident flight, it was found (in [8]) that the SDU in 9M-MRO resulted in a BFO that was initially too high at the time of log-on, followed by a simple decay over the next few minutes to a steady-state value. Regarding the log-on event at 18:25:27Z for MH370, it was noted that there was a non-zero bit error rate (BER) associated with the log-on request at that time. The associated received signal level and carrier-to-noise density ratio ( $C/N_0$ ) were also unusually low. As such, the first BFO was deemed untrustworthy.<sup>9</sup> Note that this is not the only time such a behavior was observed for the SDU in 9M-MRO. In the analysis of BFOs presented in [2], there was an outlier in the BFO (with an error of

<sup>8</sup>When viewed on a time-scale of hours as shown in Fig. 9 of [1], this behavior looks like a spike in BFO between 18:25Z and 18:28Z.

<sup>9</sup>An alternative explanation for this outlier BFO could be the unknown conditions of the aircraft between 17:22Z and 18:25Z, for instance the temperature of the SDU might have been much lower than the expected ambient temperature.

-170 Hz) found to have occurred for a SATCOM message associated with 9M-MRO in one of the 20 previous flights of 9M-MRO. This was discarded as an outlier and not presented in [2]. It too had a non-zero BER and a low  $C/N_0$  (37.6 dBHz compared to surrounding values between 41.5 and 42 dBHz). With the removal of the untrustworthy BFO logged for the first message in the sequence shown in Fig. 6, the 18:25Z log-on from MH370 was also determined to follow the simple decay trend observed in other instances.

It was noted by DST Group that if the simple BFO decay trend established after SDU power-up [8] also occurred during the MH370 SATCOM log-on event beginning at 00:19:29Z, then it would allow bounds to be established on the BFO that would have been expected for MH370 if the plane was flying level at that time. This in turn would allow the determination of bounds on the possible descent rates of MH370 during that final log-on event. In order to build confidence that the simple BFO decay trend would hold for the 00:19Z log-on event, DST Group reviewed additional Inmarsat logs for 9M-MRO, corresponding to the period 22 February to 28 February 2014. Two additional cases were identified in which 9M-MRO logged back onto an Inmarsat satellite after a sustained period of SATCOM outage. The sequence of BFOs observed in these two cases, along with the five already considered by the SATCOM working group in [8], are shown in Fig. 7, and details about each of the log-ons are given in Table II.

The periods of SATCOM outages followed by log-on events were identified from Inmarsat-provided ground-station logs by identifying sequences of three or more unsuccessful log-on interrogations to 9M-MRO (suggesting the SDU was likely powered off) followed by a log-on to the satellite system initiated from 9M-MRO at some later time. The exact duration of the power-off period for the SDU was unable to be determined from the logs, however the timestamps associated with the unsuccessful log-on interrogations and subsequent 9M-MRO initiated log-ons were used to determine bounds on the outage time as shown in the 3<sup>rd</sup> column of Table II.<sup>10</sup> Additional comments about each log-on sequence are given in the final column. Whilst some of the BERs for log-ons 1 and 6 were non-zero, the BFOs did not appear to be outliers, and were therefore considered valid. The uncertainty window in the length of the outage durations shown is roughly 60 minutes; this relates to timers in the Inmarsat satellite ground stations that result in a log-on interrogation being sent to the plane after roughly 60 minutes of SATCOM inactivity. Indeed, it is this same timing responsible for the roughly 60 minute separation between the four BTO arcs from 19:41Z to 22:41Z (e.g. [3], [1]). It is also of note that based on other evidence related to MH370 (e.g. secondary radar transponder loss of signal at 17:22Z, in which case the SDU power outage duration for log-on 7 would be approximately 63 minutes.

For log-ons 3 and 5, there were some points available about half an hour after the log-on in each case (noting the plane was

<sup>10</sup>Note that the bounds shown are derived under the assumption that the SATCOM outage was exclusively due to a power outage. It is also possible that the first part of some outages was due to a blocked line of sight to the Inmarsat satellites.

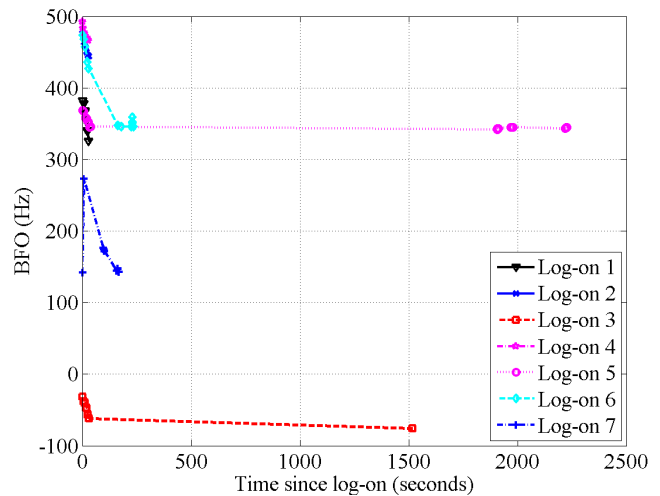


Fig. 7. Measured BFOs for 7 log-ons of 9M-MRO.

on the tarmac for the whole time) that demonstrate the BFO appears to have fully settled to a steady-state value. Log-ons 1 to 5 only lasted for less than 1 minute after the log-on request message. The results for log-ons 6 and 7, which had log-on sequences lasting a few minutes suggest a settling behavior of approximately 3 minutes duration. For all observed log-on sequences, there seems to be a simple decay of BFO over a few minutes after SDU startup and log-on request (see also Fig. 8). It is therefore possible a similar decay was occurring during the SDU log-on sequence beginning at 00:19:29Z.

In order to establish what the likely range of steady-state equivalent BFOs would have been at 00:19Z, it is useful to re-plot the curves from Fig. 7 translated vertically such that the log-on acknowledgment has 0 Hz offset. This is done in Fig. 8, which has also been zoomed in the time-axis for clarity.

For log-ons 1 to 6, the aircraft was on the ground at an airport. It has been established that the most common scenario for an on-ground-at-airport log-on after sustained power outage of the SDU is for the AES to use a different self-Doppler compensation method than was in use for log-on 7 and the log-on at 00:19Z. This is due to a lack of navigational input data from the aircraft's inertial navigation unit in the few minutes after power-up, and is referred to as the closed-loop Doppler compensation approach.<sup>11</sup>

The standard AES Doppler compensation mode when navigational data is available to the SDU is the mode used for log-on 7 and the log-on at 00:19Z. This approach was briefly described in Sec. IV-A. It used the aircraft's own estimates of its position and horizontal velocity to predict and correct the AES contributed Doppler shift. This is referred to as open-loop Doppler compensation.

For log-ons 1 to 6, we work on the premise that closed-loop Doppler compensation was used.<sup>12</sup> In that approach, the frequency of broadcast P-channel transmissions received by 9M-

<sup>11</sup>The use of closed-loop Doppler compensation in this circumstance was established in consultation with members of the MH370 Flight Path Reconstruction Group, including representatives from Thales and Inmarsat.

<sup>12</sup>Previous analysis had not considered the difference in Doppler compensation mode, yet still resulted in similar descent rates to those presented in this paper.

TABLE II  
DETAILS OF LOG-ON SEQUENCES USED FOR ANALYSIS

Identifier	Date and Timestamp of Log-on	Duration of Preceding Power Outage	Any Other Comments
Log-on 1	23rd Feb. 23:57Z	Between 381 and 442 minutes	After scheduled A1 maintenance check, some non-zero BERs
Log-on 2	26th Feb. 14:11Z	Between 295 and 354 minutes	
Log-on 3	5th Mar. 03:06Z	Between 35 and 95 minutes	
Log-on 4	6th Mar. 13:29Z	Between 43 and 103 minutes	
Log-on 5	6th Mar. 15:02Z	Between 35 and 92 minutes	
Log-on 6	7th Mar. 12:50Z	Between 228 and 288 minutes	Some non-zero BERs
Log-on 7	7th Mar. 18:25Z	Between 20 and 78 minutes	First point untrustworthy

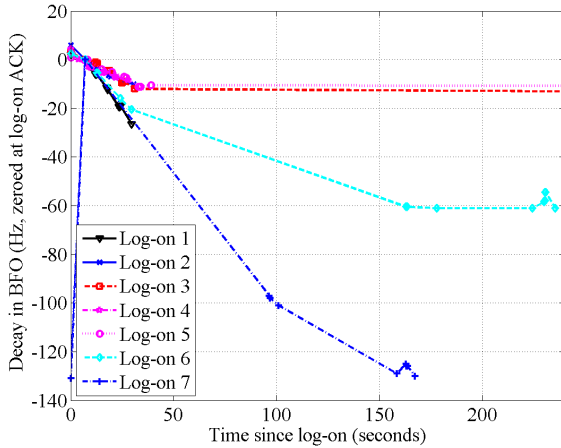


Fig. 8. Decay in measured BFOs for 7 log-ons of 9M-MRO. The decay rates for log-ons 1 to 6 have been adjusted to account for the different AES Doppler compensation mode used during these log-ons.

MRO were used to determine the AES Doppler contribution. In this approach, all of the difference between the received and expected P-channel frequency is attributed by the SDU to be caused by AES motion induced Doppler. As such, if the OCXO frequency is higher than its nominal settled value, due to incomplete temperature stabilization, the received frequency will appear lower than expected by the corresponding amount. When applying the AES Doppler compensation in this case, the transmit frequency is increased by the difference between the stable and pre-stabilized oscillator frequency (scaled to account for the difference between P-channel frequency and the R-channel transmit frequency). Moreover, the transmit frequency itself (prior to AES Doppler compensation) will already be too high by the same amount due to the incomplete temperature stabilization. The net combined effect to the BFO decay observed due the OCXO settling will therefore be doubled compared to the case where open-loop AES Doppler pre-compensation is used. Hence, the BFO decay results for log-ons 1 to 6 shown in Fig. 8 have been halved for the analysis when compared to those inferred from the raw measured BFOs presented in Fig. 7.

Recall that the two messages at 00:19:29Z and 00:19:37Z are a log-on and log-on acknowledgment respectively. From the data used to plot Fig. 8, it can be determined that the log-on acknowledgment BFOs are in the range of [0,6] Hz

lower than the log-on BFOs. Also the maximum difference between the log-on BFO and the settled BFO value is in the range [17,136] Hz. The only log-on that doesn't appear to be approaching a settled value in this range is log-on 1, for which it is anticipated that if more data points were available several minutes later, as was the case for log-ons 3, 5, 6 and 7, the settling behavior would be similar to that for log-ons 6 and 7, bearing in mind the outage duration for log-on 1 is probably at least 381 minutes, and the BFO already dropped substantially in the first 30 seconds after log-on.

#### B. Summary of SDU Startup Effects on the BFO

It is likely that the SDU in 9M-MRO lost power sometime after 00:11Z, and then regained power (presumably due to Auxiliary Power Unit (APU) startup) prior to 00:19Z, leading to the SDU making the log-on attempt at 00:19:29Z. Previous such events for 9M-MRO have shown that this results in an initially too-high BFO for the log-on message, followed by a simple decay characteristic to reach a steady-state BFO after several minutes. In order to establish the relevance of the BFOs at 00:19:29Z and 00:19:37Z to the motion (including descent rate) of MH370 at those times, this section has established bounds for the likely range of steady-state equivalent BFOs at those times. This was done by analyzing BFOs from seven previous SDU start-up events for a period of several minutes. The specific results obtained suggest that:

- 1) The recorded BFO for the 00:19:29Z log-on was between 17 and 136 Hz higher than it would have been if the OCXO in the SDU was in a steady state at that time;
- 2) The recorded BFO for the 00:19:37Z log-on acknowledgment message was between 17 and 130 Hz higher than it would have been if the OCXO in the SDU was in a steady state at that time.

## VI. BOUNDING THE DESCENT RATES OF MH370

The results from Sections IV and V can be combined to provide bounds on the descent rate of MH370 implied by the BFOs from the two last SATCOM messages for the flight. In this section, it is shown how this is done for two different possibilities that could explain the attempted SATCOM log-on from 9M-MRO at 00:19Z. In [3] the most likely cause of this log-on was stated to be a power interruption resulting from insufficient fuel and subsequent engine flameout. It could

also have been due to a temporary software failure, a loss of systems providing critical input to the SDU, or a loss of the SATCOM link due to aircraft attitude being such that the line-of-sight to the satellite is blocked. If it was indeed a power interruption to the SDU caused by loss of fuel and subsequent reboot using the APU, the SDU would be without power for about one minute. In this case, the results of Sec. V-B need to be considered when interpreting the last two BFOs.<sup>13</sup> If on the other hand, the power loss was momentary (resulting in a reset of the SDU) or if the temporary SATCOM outage leading to the log-on request was due to one of the other listed reasons, there would be no “warm-up drift” to consider, so the results of Sec. V would not need to be applied. Both cases are considered separately in the following two sub-sections, and then combined overall bounds are presented.

#### A. Hypothesis 1: SATCOM outage due to insufficient fuel

In the event that the SDU log-on at 00:19:29Z was due to engine flame-out, followed by a restart of the SDU using power from the APU, the SDU outage preceding the log-on would have lasted about one minute. This would result in some cooling of the OCXO in the SDU. The likely effect of the SDU startup and consequent ‘warm-up drift’ of the OCXO is summarized in Sec. V-B. Table III presents the recorded BFOs and bounds on the adjusted BFOs to remove these effects of warm-up drift. In the last column of the table, the bounds are extended taking into account the BFO noise bounds of  $[-28, +18]$  Hz established in Sec. III-A.

As established in Sec. IV-B, the recorded BFO would be roughly 1.7 Hz lower for every 100 fpm of descent rate. As such, depending on whether the plane was tracking South or North (minimum or maximum expected BFOs, respectively, see Sec. IV-C), bounds on the descent rate of MH370 at the times of transmission corresponding to the last 2 BFOs can be determined by subtracting the values given in the rightmost column of Table III from the expected BFO values for level flight tracking South or North, dividing the result by 1.7 Hz and multiplying by 100 fpm. The expected BFO for a south track is approximately 260 Hz, whilst for a north track it is close to 280 Hz. Using these numbers, it is straightforward to obtain the bounds shown in Table IV. Note that the bounds have been rounded to the nearest 100 fpm. Looking at all values in the table, it can be concluded that irrespective of ground track angle and for assumed ground speeds less than approximately 500 kts, under Hypothesis 1, MH370 would have been descending at between 3,900 and 14,800 fpm at 00:19:29Z, and just 8 seconds later at between 14,800 and 25,300 fpm. These descent rates are consistent with simulations of an uncontrolled phugoid descent reported in [7].

#### B. Hypothesis 2: SATCOM outage due to some other reason

It is still of interest to determine bounds on the descent rates under the alternate hypothesis that something else not

associated with a power outage led to the SDU-initiated log-on event at 00:19Z. In this case, the warm-up drift would not apply. Therefore, the recorded BFOs can be treated normally (though still subject to BFO noise). In this case, the BFO bounds are as set out in Table V. Using these bounds, the lower and upper descent rates can be calculated again, resulting in the bounds presented in Table VI. It can be concluded from the table that the recorded BFOs indicate that at 00:19:29Z the plane was descending at between 2,900 fpm and 6,800 fpm. At the time of the last SATCOM message, it can be concluded under Hypothesis 2 that the descent rate would have been between 13,800 fpm and 17,600 fpm.

#### C. Summary of Descent Rate Bounds

Combining the descent rate bounds for Hypotheses 1 and 2, outer bounds can be established for the descent rate of MH370 at 00:19:29Z and 00:19:37Z. Specifically, regardless of ground track angle and for ground speeds less than approximately 500 kts, accounting for possible BFO noise, and regardless of whether the SATCOM outage between 00:11Z and 00:19Z was due to an SDU power outage or another reason, the outer bounds on the possible descent rates at the times of the last two SATCOM messages from MH370 are given in Table VII.

#### D. Estimated Downwards Acceleration

When interpreting the bounds presented in Table VII, the different conditions under which each bound was derived need to be considered. For instance the lowest rate at 00:19:29Z was derived assuming no period of SDU outage and a southwards track, whereas the highest descent rate for 00:19:37Z was derived assuming a one minute outage of the SDU with the maximum BFO decay observed in the previous 7 SDU outage events for 9M-MRO. Hence, it would not be reasonable to say that the plane could have been descending at 2,900 fpm at 00:19:29Z and 25,300 fpm 8 seconds later, which would imply a downwards acceleration on the order of 2,800 fpm per second. Whilst it is not possible to determine a precise acceleration value, a rough approximation of the average descent rate over those 8 seconds could for instance be taken using the mid-points of the bounds at each time, which would result in an average downwards acceleration of 10,700 fpm in 8 seconds or around 1,300 fpm per second, which equates to around  $6.7 \text{ ms}^{-2}$  or  $0.68g$ , where  $g$  denotes Earth’s gravitational constant, which is approximately  $9.8 \text{ ms}^{-2}$ . It is straightforward to see that other reasonable methods of estimating the downward acceleration (such as taking the difference between the two minimum descent rates or the two maximum descent rates at 00:19:29Z and 00:19:37Z) would yield similar acceleration values.

## VII. CONCLUSION

This article has discussed in detail all known factors that could have influenced the BFOs associated with the last two SATCOM messages received from MH370 at 00:19Z. Lower and upper bounds on its descent rate at this time

<sup>13</sup>Given the relatively short duration of the power-loss under Hypothesis 1, it is possible that the extent of BFO decay due to OCXO warm-up would be less than shown for log-ons 1 to 7. This is essentially covered within Hypothesis 2, which considers a momentary SATCOM outage.



TABLE III  
THE LAST TWO BFOs FOR MH370 UNDER HYPOTHESIS 1

Timestamp	Recorded BFO (Hz)	BFO Range if Start-Up Drift Removed (Hz)	Extended Range Considering BFO Noise (Hz)
00:19:29Z	182	[46, 165]	[28, 193]
00:19:37Z	-2	[-132, -19]	[-150, 9]

TABLE IV  
MH370 DESCENT RATES AT 00:19Z 8 MARCH 2014 UNDER HYPOTHESIS 1

Timestamp	Min. Desc. Rate, South Track	Min. Desc. Rate, North Track	Max. Desc. Rate, South Track	Max. Desc. Rate, North Track
00:19:29Z	3,900 fpm	5,100 fpm	13,600 fpm	14,800 fpm
00:19:37Z	14,800 fpm	15,900 fpm	24,100 fpm	25,300 fpm

TABLE V  
THE LAST TWO BFOs FOR MH370 UNDER HYPOTHESIS 2

Timestamp	Recorded BFO (Hz)	BFO Bounds With Noise (Hz)
00:19:29Z	182	[164, 210]
00:19:37Z	-2	[-20, 26]

were then derived.<sup>14</sup> The downwards acceleration over the 8 second interval between these two messages was found to be approximately 0.68*g*. The derived bounds and approximate downwards acceleration rate are consistent with simulations of an uncontrolled descent near the 7<sup>th</sup> arc as reported in [7]. This suggests that 9M-MRO should lie relatively close to the 7<sup>th</sup> BTO arc.

At the time of submission of this paper, MH370 had not been located. The search was in indefinite suspension pending ‘credible new evidence pointing to a specific location for the plane’. Further efforts in data analysis and in drift modeling of located MH370 debris have been conducted and reported (e.g. [9], [10], [7]). Note that the descent analysis from this paper was used in [7] when considering the width of any potential future search area. Some recent news reports have suggested a US underwater exploration firm named Ocean Infinity may soon be launching a renewed search effort for MH370. The reader should refer to the Australian Transport Safety Bureau website [11] and the general media for updates.

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<sup>14</sup>The conclusions regarding descent rate have been made taking into account BFO noise bounds, all reasonably feasible ground tracks and speeds, and possible OCXO warm-up drift. It has been implicitly assumed that there were no otherwise unknown factors that could have affected the last two BFOs.

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nology Group, Australia.

TABLE VI  
MH370 DESCENT RATES AT 00:19Z 8 MARCH 2014 UNDER HYPOTHESIS 2

Timestamp	Min. Desc. Rate, South Track	Min. Desc. Rate, North Track	Max. Desc. Rate, South Track	Max. Desc. Rate, North Track
00:19:29Z	2,900 fpm	4,100 fpm	5,600 fpm	6,800 fpm
00:19:37Z	13,800 fpm	14,900 fpm	16,500 fpm	17,600 fpm

TABLE VII  
RANGE OF POSSIBLE MH370 DESCENT RATES AT 00:19Z 8 MARCH 2014

Timestamp	Min. Desc. Rate	Max. Desc. Rate
00:19:29Z	2,900 fpm	14,800 fpm
00:19:37Z	13,800 fpm	25,300 fpm